B.Sc. Part III (Hons) 7th Paper

Diff. EQNs. (CONTD.)

LDEVC

Q. Solve $x^2 y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$.

Soln The given equation

$$x^2 y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$$

$$\Rightarrow y_2 - \frac{2(1+x)}{x} y_1 + \frac{2(1+x)}{x^2} y = x \quad \text{---(1)}$$

which is of the form

$$y_2 + Py' + Qy = R.$$

Here, $P = -\frac{2(1+x)}{x}$, $Q = \frac{2(1+x)}{x^2}$, $R = x$ ---(2)

Now, $P + Qx = -\frac{2(1+x)}{x} + \frac{2(1+x)}{x} = 0.$

$\Rightarrow$ $u = x$ is a part of CF of (1). ---(3)

Let $y = uv$ be the general soln of (1)

$$\Rightarrow \boxed{\frac{d^2v}{dx^2} + \left(P + \frac{2}{u}\frac{du}{dx}\right)\frac{dv}{dx} = \frac{R}{u}}$$

$$\Rightarrow \frac{d^2v}{dx^2} + \left[-\frac{2(1+x)}{x} + \frac{2}{x}\frac{d(x)}{dx}\right]\frac{dv}{dx} = \frac{x}{x}$$

$$\Rightarrow \frac{d^2v}{dx^2} + \left(-\cancel{\frac{2}{x}} - \frac{2x}{x} + \cancel{\frac{2}{x}}\right)\frac{dv}{dx} = 1$$

$$\Rightarrow \frac{d^2v}{dx^2} - 2\frac{dv}{dx} = 1 \quad \text{---(4)}$$

Put $\dfrac{dv}{dx} = z \Rightarrow \dfrac{d^2v}{dx^2} = \dfrac{dz}{dx}$

(4) $\Rightarrow \dfrac{dz}{dx} - 2z = 1 \Rightarrow \dfrac{dz}{dx} = 1+2z$

$\Rightarrow \dfrac{dz}{1+2z} = dx$

$\Rightarrow \dfrac{2\,dz}{1+2z} = 2\,dx$ integrating, we get

$\Rightarrow \displaystyle\int \dfrac{2\,dz}{1+2z} = 2\int dx$

$\Rightarrow \log(1+2z) = 2x + \log k$

$\Rightarrow 1+2z = k e^{2x}$

$\Rightarrow 2z = k e^{2x} - 1 \Rightarrow 2\dfrac{dv}{dx} = k e^{2x} - 1$
$\left[\because z = \dfrac{dv}{dx}\right]$

$\Rightarrow 2\,dv = k e^{2x}\,dx - dx$

integrating we get

$\Rightarrow 2v = k\dfrac{e^{2x}}{2} - x + 2k_1$

$\Rightarrow v = k\dfrac{e^{2x}}{4} - \dfrac{x}{2} + k_1$ —— (5)

Hence, $y = uv$ where $u = x$ and v is given by eq (5) is the general solution.